

Preprint LNPI-1099, 1985

# ELECTROMAGNETIC FIELDS PROVIDING CHARGE SCREENING AND CONFINEMENT IN TWO-DIMENSIONAL MASSLESS ELECTRODYNAMICS\*

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## Abstract

The charge screening, confinement of fermion quantum numbers and the chiral condensate formation in two-dimensional QED is studied in details. It is shown that charge screening and confinement of fermion number in two-dimensional QED is due to an appearance of gauge fields which nullify the Dirac determinant  $D(A)$ . An appearance of the fields of another type but with the same property yield the chiral condensate formation. In addition, these second type fields ensure the "softness" of the charge screening in a process which is analogous to the  $e^+e^-$  annihilation.

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\*The brief version of this paper has been published in *Yadernaja Fizika: Yad. Fiz.* 41 (1985) 534 (in Russian).

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# 1 Introduction

The conventional point of view [1] connects quark confinement with a linear potential which is expected for two static quarks inserted in the vacuum of pure gluodynamics. It is usually assumed that the colour field of the quarks interacting with non-perturbative gluon vacuum fluctuations shrinks into the tube. This leads to a linear growth of the energy with the distance between quarks and confines them. The situation could drastically change, however, when light quarks are introduced in the theory. Here the string between static quarks instantly tears and created quark pairs screen the linear potential of the pure gluodynamics. This means that the specific properties of QCD with light quarks are determined by configurations of gluon fields rather different from the configurations providing confinement in pure gluodynamics.

The simple example of the phenomenon discussed is the well-known Schwinger model presenting the quantum electrodynamics of massless fermions (quarks) in two dimensions [2,3]. Here the original Coulomb potential is indeed proportional to the distance but the interaction with massless quarks screens it out. As the result, one can observe in QED<sub>2</sub>

- i) the absence of charged (coloured) states [2–4];
- ii) confinement of quark quantum numbers (chirality, fermion number [5], flavour for multi-flavour QED<sub>2</sub> [4,6]);
- iii) the chiral vacuum condensate [3,6,7];
- iv) the independence of quark jets and "soft" hadronization [8,9].

The above features of QED<sub>2</sub> seem to be similar to that expected in QCD.

In this paper we just investigate the electromagnetic (e.m.) fields which provide the above features of QED<sub>2</sub>. We show that these are fields for which the Dirac operator determinant  $D(A)$  vanishes. We divide the above fields in two types to be, respectively, A and B.

The fields of the first (A) type are necessary generated in QED<sub>2</sub> by charged systems. They are responsible for charge confinement in QED<sub>2</sub>. In order to prove this we consider the evolution operator  $S(T)$  of the system in the quark representation. This operator is the sum over all physical states

$$S(T) = \sum_h e^{-iE_n T} |\Psi_n \rangle \langle \Psi_n|, \quad (1)$$

$E_n$  being the energy of the state. Matrix elements of the operator  $S(T)$ , Eq.(1), can be represented as functional integrals over the fields with boundary values fixed at initial and final momenta of time. In the Coulomb gauge which we shall use in the paper we have:<sup>1</sup>

$$S(T) = \int DA_0 \exp \left[ -i \int_0^T \frac{F_{\mu\nu}^2}{4} d^2x \right] \int_{\Psi_{in}(0)}^{\Psi_{out}(T)} D\Psi D\bar{\Psi} \exp \left[ i \int_0^T \bar{\Psi} (i\hat{\partial} + g\hat{A}_0) \Psi d^2x \right]. \quad (2)$$

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<sup>1</sup>This gauge is convenient since here are no unphysical degrees of freedom. This gives possibility to present a clear physical interpretation of all phenomena considered

For QED<sub>2</sub> we have constructed  $S(T)$ , in ref.[10].

Matrix elements of the evolution operator determine the probability amplitudes to have any quark configuration in any physical state. The charge confinement means that such amplitudes must vanish for the configurations where some charge (quark  $q$ ) is situated far from other compensating charges (antiquark  $\bar{q}$ ). We shall demonstrate that this is the case in QED<sub>2</sub> and the amplitude to find quark on distance  $r$  from antiquark falls as  $r^{-2}$ . Investigating the structure of the integral, Eq.(2), one can discover the reason for such a dependence: the self-consistent field  $\bar{A}$  of the distantly situated quark annihilation the quark determinant  $D(A)$ . This is the field of the class A. Since the amplitude is proportional to  $D(\bar{A})$  it naturally vanishes at large  $r$ . The explicit proportionality to  $D(A)$  appears in Eq.(2), after integration over fermion fields. We have [10]:

$$S(T) = \int DA_0 \exp \left[ -i \int_0^T \frac{F_{\mu\nu}^2}{4} d^2x \right] S_{ext}(A_0). \quad (3)$$

Here  $S_{ext}(A_0, t) \sim D(A_0)$  is the evolution operator in the given external field  $A_0$  (explicit form see in Sec.2). Its matrix elements are determined by chosen quark configurations and are expressed in terms of the fermion Green functions in the external e.m. field  $A_0$  (Eq.(5)).

Now we see that if for some quark configuration a self-consistent field  $\bar{A}_0$  nullifies  $D(A)$  then this configuration is prohibited; it cannot appear in evolution of the system. It is this mechanism which forbids existence of charged states in QED<sub>2</sub>. As we have stated the field of every charged quark creates the field, which makes determinant to vanish. The field strength of the discussed field behaves as  $1/x$  for large distances due to the effect of the vacuum polarization.

The fields with  $D(A) \rightarrow 0$  always create new quark pairs.<sup>2</sup> These pairs screen original charges. If the total screening is not achieved, a new configuration also corresponds to  $D(\bar{A}) \rightarrow 0$  and the process repeats. It is well known that  $D(A)$  determines vacuum polarization effects. Hence, the described processes represent the mechanism of charge screening by means of the vacuum polarization.

As it had been noted by many authors [6,11,12], the screening mechanism of QED<sub>2</sub> reminds in some aspects the well-known Higgs one. However, there is an essential difference between them. The QED<sub>2</sub> models contain no charged condensate because any configuration with separated charges cannot be present in the evolution operator  $S(T)$ . Therefore, there are no charged complexes in the physical vacuum. We had seen this fact investigating the quark structure of QED<sub>2</sub> vacuum states [7]. Charge screening is not statistical (in average) as it happens in the Higgs case, but is the exact one. The wave functions of excitations are here the eigenfunctions of the charge operator with vanishing eigenvalues. It is just the spectrum of the type expected for the real world [1].

The fields of the second class (B) govern chiral properties of the model. They also make  $D(A) \rightarrow 0$ , but decrease with a space distance faster than  $x^{-1}$ . They can be characterized

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<sup>2</sup>In fact,  $|D(A)|^2$  is the probability of the process where no quark pairs are produced.

by a non-zero value of the integral

$$Q_{top}(T) = -\frac{g}{2\pi} \int_0^T \int d^2x E(x, t) = -\frac{g}{4\pi} \int_0^T \int d^2x \varepsilon_{\mu\nu} F^{\mu\nu}(x), \quad (4)$$

which is the two-dimensional winding number [13]. The  $B$ -fields also inevitably produce  $q\bar{q}$ -pairs. As we shall see  $2|Q_{top}|$  particles of these pairs necessarily have very small momenta  $p \sim 1/V$ , being considered in the finite space volume  $V$  ( $V \rightarrow \infty$ ). They are delocalized over the whole volume. Such particles do not contribute to any local quantities and are, in fact, unobservable. In the  $V \rightarrow \infty$  limit, the non-conservation of chirality  $K$  (which follows from the Adler-Bell-Jackiw (ABJ)-anomaly [14]) can be interpreted in QED<sub>2</sub> just as a generation of delocalized particles with small momenta  $p \sim 1/V$  by  $B$ -fields.

The  $B$ -fields with integer  $Q_{top}$  play an important role in the formation of quark condensates. For  $Q_{top} = N$  exactly  $N$  pairs  $\bar{q}_R q_L$  (or  $q_R \bar{q}_L$  for  $N < 0$ ) are generated by such a  $B$ -type field. The chiral condensates of QED<sub>2</sub> with one and several flavours [2,6] are formed by similar chiral complexes [7] (Sec.3).

We shall see in Sec.3 that  $B$ -fields prevent a change of chirality for developing quark configurations. Owing to this property the exchange of chirality between hadrons and the chiral condensate is impossible. As a result, chirality of hadrons is conserved and appears to be the definite fixed quantity ( $K = 0$ ).

In some aspects the properties of  $B$ -fields are similar to those of instanton fields [15] in QCD. However there are essential differences between them, as it is discussed in Sec.2.

The second main topic of this article is the hadronization process investigated in Sec.4. We consider there the production of a  $q_R \bar{q}_L$ -pair by an external source. This, widely discussed in QED<sub>2</sub>, process [3,8,9] is an analogue of the  $e^+e^-$ -hadrons reaction. It is also governed by a field with  $D(A) \rightarrow 0$  (B-type). Such a field produces just that one  $\bar{q}_R q_L$ -pair with finite momenta ( $p \sim m$ ) which screens charges and chiralities of the both jets. Simultaneously, the same field creates another  $q_R \bar{q}_L$ -pair with the vanishing total momenta which joins the vacuum condensate.

Thus the screening of charges and chiralities for both jets is finished at the finite time  $t \sim m^{-1}$  ( $m = g^2/\pi$  is the hadron mass), when  $Q_{top}(T)$  becomes of the order of unity. At larger times  $t > 1/m$  we have only neutral independent jets with  $K = 0$ . From these times, there begins a rapid growth (proportional to  $t^2$ ) in the number of quarks with momenta  $p \gg m$ . At times  $t \sim p/m^2$  their distribution is transformed into the hadron distribution of the parton model ( $1/p$ ). The leading pair of quarks loses its energy, and after a time  $t \sim p_{in}/m^2$  participates in the formation of the hadrons. The produced hadrons with  $p < p_{in}$  become spatially separated from the leading quarks of the initial pair. Quarks exist outside hadrons only on the light cone.

## 2 Properties of fields where $D(A) \rightarrow 0$

EM fields which make the determinant  $D(A) \rightarrow 0$  correspond to those quark configurations which are prohibited for evolution and production. In order to prove this statement let us

consider the evolution operator  $S(T)$  for a finite time interval  $T$ . As it was shown in Ref.[10], the operator  $S(T)$  can be represented as a functional integral over  $A$ . In the Coulomb gauge ( $A_1 = 0$ ) we have Eq.(2) for  $S(T)$ .

The integration in Eq.(2) goes over all EM fields  $A_0$  in the time interval  $0 \leq t \leq T$  including end points  $t = 0$  and  $t = T$ . The evolution operator for massless fermion in an external field  $A_0$  is equal to [10]

$$\begin{aligned}
S_{ext}(T) &= D(A_0) \exp \left\{ \int dx dx' \sum_{i=R,L} \left[ a_i^+(x) G_i^{(T)}(xT, x'0) a_i(x') \right. \right. \\
&\quad + b_i(x) G_i^{(T)}(x, 0; x', T) b_i^+(x') - a_i^+(x) G_i^{(T)}(x, T - \varepsilon; x'T) b_i^+(x') \\
&\quad \left. \left. - b_i(x) G_i^{(T)}(x, \varepsilon; x', 0) a_i(x') \right] \right\}; \\
\varepsilon &\rightarrow +0.
\end{aligned} \tag{5}$$

Here  $D(A_0)$  is the determinant of the Dirac equation in the external field  $A_0$ ;  $a_{RL}^\pm(x)$ ,  $b_{RL}^\pm(x)$  ( $a^- \equiv a$ ) are the creation and annihilation operator for right ( $R$ ) and left ( $L$ ) quarks and antiquarks:

$$\Psi_i(x) = a_i(x) + b_i^+(x), \quad i = R, L, \quad \Psi_{R,L}(x) = \frac{1}{2}(1 \pm \gamma_5)\Psi(x). \tag{6}$$

The Green functions  $G_i$  in an external field have the form [10]

$$\begin{aligned}
G_i^{(T)}(x, t; x', t') &= G_i^{(0)}(x, t; x', t') \exp \left\{ ig \int_0^T dt_1 \int dx_1 A_0(x, t) \right. \\
&\quad \left. \times \left[ G_i^{(0)}(x, t; x'_1, t_1) - G_i^{(0)}(x', t'; x_1, t_1) \right] \right\}, \\
G_i^{(0)}(x, t; x', t') &= [2\pi i(t - t' \mp (x - x) - i0 \text{ sign } (t - t'))]^{-1},
\end{aligned} \tag{7}$$

$g$  is the dimensional coupling constant of QED<sub>2</sub>.

For any processes, matrix elements of  $S(T)$  are determined by quark configurations in the initial  $(a_i(x), b_i(x))$  and final  $(a_i^+(x), b_i^+(x))$  states. These matrices appear in Eq.(3) as coefficients in front of corresponding numbers of  $a^\pm$ ,  $b^\pm$ -operators. To obtain matrix elements we must expand the exponential in Eq.(5). Then the integral, Eq.(3), will be of the Gauss--type one. Its value can be obtained substituting the saddle-point field  $\bar{A}_0$ , in every term of the integrand.

Hence, if  $D(\bar{A}_0) \rightarrow 0$ , the configuration discussed does not contribute in the norm of any physical state vector:

$$\Psi(T) = S(T)\Psi_0. \tag{8}$$

It does mean that this configuration cannot appear as a result of evolution of the system. If charged states exist, then distantly situated charges would be represented in matrix elements of the  $S(T)$ -operator with a finite probability. There are no charged states when all such matrix elements decrease with space distance  $r$  between charges. As we shall see, in QED<sub>2</sub> all the above phenomena are due to fields providing  $D(A) \rightarrow 0$ , at  $r \rightarrow \infty$ .

In this section we discuss general properties of the fields  $D(A) \rightarrow 0$ . We take here fields as external ones. Self-consistent fields of QED<sub>2</sub> considered in the next section, are of the  $D(A) \rightarrow 0$  type, too.

The Dirac determinant for massless fermions in an external field has the following form [10]:

$$\ln D(A) = -\frac{m^2}{2} \int \frac{dp}{2\pi} \int_0^T dt dt' E(p, t) E(-p, t) \frac{e^{-i|p||t-t'|}}{2|p|}. \quad (9)$$

$m^2 = g^2/\pi$ ,  $E(x, t) = -\partial A_0/\partial x$  is the field strength in the Coulomb gauge.

All the field configurations discussed correspond to finite field strength. Then  $D(A)$  can vanish only because of the divergence of the integral in Eq.(9) at  $p \rightarrow 0$ . Therefore, we shall consider two types of the field  $D(A) \rightarrow 0$

$$(A) \ E(p, t) \rightarrow \text{sign} p f_1(t), \quad (B) \ E(p, t) \rightarrow f_2(t), \ p \rightarrow 0, \quad (10)$$

where  $f_i(t)$  are some functions of  $t$ . These fields have different asymptotics for  $x \rightarrow \infty$ . The field strength for the A-type fields decreases as  $1/x$ , while  $E$  for the B-type ones decreases more rapidly.

Evaluating a divergent part of the integral, Eq.(9), we obtain the following formulae for the  $D(A)$  using space volume  $V$  as an infrared regularization:

$$D(A) = 1/V \beta^{2(T)}, \quad \beta(T) = \frac{g}{2\pi} \int_0^T dt \int dx V.P. \int \frac{dy E(y, t)}{x - y} \quad (11)$$

for the A-type fields and

$$D(A) = 1/V^{|Q_{top}(T)|}, \quad Q_{top}(T) = -\frac{g}{2\pi} \int_0^T dt \int dx E(x, t) \quad (12)$$

for the B-type.  $Q_{top}$  is the "topological charge", of the field (see Sec.1).

The main contribution to the norm of state vectors for the fields  $D(A) \rightarrow 0$  comes from the particles with small momentum  $p \sim 1/V$ . Indeed, since the norm of wave functions in Eq.(8) is conserved, the small factor  $D(A)$  should be compensated by the contribution of quark configurations expanded over the large volume  $V$ . Particles with small momenta  $p \sim 1/V$  represent just such configurations. So the probability of no pair creation in  $D(A) \rightarrow 0$  fields is suppressed by the factor  $\sim 1/V$  to be compared with the probability of configurations with quark pairs.

In order to make explicit the role of quarks with small momenta for A and B fields, let us consider the number  $n_R(p, t)$  of quarks produced by such external field in the fermion vacuum:  $\Psi_0(t) = \exp(-iH_{ext}(t))|0\rangle$ . The simplest way to calculate  $n_R(p, t)$  is to use the bosonization procedure. For the fields of the B-type these calculations have been performed in detail in Sec.3 of Ref.[9]. To find  $\Psi_0(t)$ , one must solve the Schrödinger equation in an arbitrary  $A_0$  external field taken in the Coulomb gauge. For this aim we express  $\Psi(x)$ ,  $\Psi^+(x)$  operators by means of bosonization forms [9] and calculate  $n_R(p, t)$  as follows:

$$n_R(p, t) = \langle \Psi_0(t) | a_R^\dagger(p) a_R(p) | \Psi_0(t) \rangle = \int dx dy e^{ip(x-y)}$$

$$\begin{aligned} \langle \Psi_0(t) | \Psi_R^+(x) \Psi_R(y) | \Psi_0(t) \rangle &= \int \frac{dxdy}{2\pi i} \frac{e^{ip(x-y)}}{y-x-i0} \\ &\times \exp\{2\pi i[\alpha(x,t) - \alpha(y,t)]\}. \end{aligned} \quad (13)$$

We use the normalization condition to be  $\{a^+(p), a(p')\} = V\delta_{p,p'}$ . In this case we have

$$\begin{aligned} \alpha(x,t) &= -\frac{g}{2\pi} \int_0^t dt_1 \int dx_1 \Theta(t-x-t_1+x_1) E(x_1, t_1) = \int \frac{dk}{2\pi} \frac{e^{-ik(t-x)}}{k} \Phi_R(k, t) \\ \Phi_R(k, t) &= g/2\pi \int dt_1 \int dx_1 e^{ik(t_1-x_1)} E(x_1, t_1). \end{aligned} \quad (14)$$

For small  $k$  we have for A and B types respectively:

$$\Phi_R(k, t) = i \text{sign} k \beta(t), \quad \Phi_R(k, t) = -Q_{top}(t). \quad (15)$$

Substituting Eqs. (14) and (15) to Eq.(13) and taking into account the relation

$$\frac{1}{2\pi(y-x-i0)} = \int \frac{dk}{2\pi} e^{-ik(y-x)}, \quad (16)$$

we obtain the following formulae:

$$n_R(p, t) = \frac{sh^2\pi\beta(t)}{p} \quad (A), \quad n_R(p, t) = \frac{\sin^2 \pi Q_{top}(t)}{p} \quad (B). \quad (17)$$

Thus, for non-integer  $Q_{top}$  both A and B type fields produce an infinite number of  $q\bar{q}$ -pairs with small momenta  $p \rightarrow 0$ . In the case of integer  $Q_{top}$ , Eq.(17) is not defined when  $p \rightarrow 0$ . To obtain the correct expression for  $n_R(p, t)$  in the above case, let us consider the system in a finite space volume  $V$ . Then, Eq.(13) must be changed as

$$\begin{aligned} n_R(p_n, t) &= \int_{-V/2}^{V/2} dxdy e^{ip_n(x-y)} G_R^{(0)}(x-y) \exp \left[ \frac{2\pi}{V} \sum_{k_n > 0} \frac{\Phi_R(k_n, t)}{k_n} \right. \\ &\times \left. \left( e^{-ik_n(t-x)} - e^{ik_n(t-x)} + e^{ik_n(t-y)} - e^{-ik_n(t-y)} \right) \right], \end{aligned} \quad (18)$$

$p_n, k_n = 2\pi n/V$  to be an integer. Furthermore, in the considered case the free Green function  $G_0$  has the following form:

$$G_R^{(0)}(x-y) = \frac{1}{V} \sum_{p_n > 0} e^{ip_n(x-y)} = \frac{\exp(2\pi i(x-y)/V)}{V[1 - \exp(2\pi i(x-y)/V)]}. \quad (19)$$

Due to the  $\{a^+(p), a(p)\} = V$  relation, to obtain the number of particles  $N_R(p_n, t)$  with the given momentum  $p_n$  we must divide  $n_R(p_n, t)$  by volume  $V$ . Finally, we substitute instead

of  $\Phi_R(k_n, t)$  in Eq.(18) its value at  $k_n = 0$  equal to  $Q_{top}(t)$  and obtain:

$$N_R(p_n, t) = \frac{1}{V^2} \int_{-V/2}^{V/2} dx dy \frac{e^{ip_n(x-y)} e^{2i(x-y)/V}}{1 - \exp 2i(x-y)/V} \quad (20)$$

$$\times \exp \left[ -\frac{2\pi Q_{top}}{V} \sum_{k_n > 0} \frac{1}{k_n} \left( e^{-ik_n(t-x)} + e^{ik_n(t-y)} - \text{h.c.} \right) \right].$$

Summing up the series in the exponential factor we find

$$N_R(p_n, t) = \int_{-V/2}^{V/2} \frac{dx dy}{V^2} e^{ip_n(x-y)} \frac{\exp[2\pi i/V(x-y)] \exp[2\pi i Q_{top}(x-y)/V]}{1 - \exp[2i(x-y)/V]}. \quad (21)$$

Hence, for  $Q_{top} < 0$  we get:

$$N_R(p_n, t) = \begin{cases} 0 & p_n \geq |Q_{top}| 2\pi/V \\ 1 & p_n = 0, 2\pi/V, \dots, 2\pi(|Q_{top}| - 1)/V. \end{cases} \quad (22)$$

We see that B-type field with an integer  $Q_{top} < 0$  produces exactly  $|Q_{top}|$  of  $R$ -quarks with vanishing momenta  $0, 2\pi/V, \dots, 2\pi/V, (|Q_{top}| - 1)2\pi/V$ . Simultaneously, the same field produces the equal number  $|Q_{top}|$  of  $L$  antiquarks with identical momenta. For  $Q_{top} > 0$  quarks and antiquarks change places. Namely, in this case,  $Q_{top}$  of the  $L$  quarks and  $Q_{top}$  antiquarks are produced.

The chiral properties of fermion systems in an external field were investigated in Ref.[9] just for the case of B-fields. The total chirality is changed according to the ABJ-anomaly [14] as

$$K(t) - K(0) = 2Q_{top}(t). \quad (23)$$

Here  $K(t) = \int dx j_0^5(t, x)$  is the total chirality at time  $t$ ,  $j^5(x)$  being the axial current density.

The explicit calculations [9] for  $j_0^5(x, t)$  show that in the case of a finite volume system, there appear two different components in the density of chirality when a B-field acts. The first component stays finite at  $V \rightarrow \infty$ . The second component contains terms  $\sim 1/V$  vanishing with  $V \rightarrow \infty$  but giving a nonzero result when they are integrated over  $V$ . These terms appear due to the discussed above phenomenon: particles with small momenta  $p \sim 1/V$  are necessarily produced by B-fields. So the corresponding part of the total chirality  $-2(Q_{top})$  becomes delocalized over the volume  $V$ . The localized part being formed by contributions of quarks-antiquarks with finite momenta, is not conserved and has to obey just Eq.(23). But the total chirality (including the delocalized part) is conserved in the volume  $V$ .<sup>3</sup> For  $V \rightarrow \infty$  the described phenomenon represents the physical interpretation of the ABJ-anomaly for QED<sub>2</sub> in the Coulomb gauge.

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<sup>3</sup> It means that the equation for the divergence of axial current (ABJ-anomaly) changes for the case of finite volume as follows (Ref.[9]):  $ip_n \rho(p_n, t) + j(p_n, t) = -g/2\pi E(p_n, t)$ ,  $n \neq 0$ ,  $j(p_n \neq 0, t) = \dot{K} = 0$ . with  $(p_n = 2\pi n/V)$ . So the total chirality being the particle number  $(N_R - \bar{N}_R) - (N_L - \bar{N}_L)$ , is conserved for the fermion system in the external fields.



Moreover, this effect provides both the ABJ-anomaly and QED<sub>2</sub> chiral condensates. The matter is the following. When  $Q_{top}$  is an integer number it is  $(Q_{top})$  of  $q_R\bar{q}_L(q_L\bar{q}_R)$  pairs to be delocalized with  $p \sim 1/V$ . In the case with an external field when  $Q_{top}$  is non-integer, in a coherent state with the distribution of Eq.(17) a non-integer number of quarks is delocalized. On the other hand, QED<sub>2</sub> models have chiral vacuum condensates only when  $|Q_{top}|$  is integer for essential self-consistent fields (see next Sec.). This situation exists in the Schwinger model where  $|Q_{top}| = 1$  and the condensate is formed mainly by  $q_R\bar{q}_L(q_L\bar{q}_R)$  complexes. For the QED<sub>2</sub> model with many electron flavours [4,6] there are no condensates without any symmetry between flavours. But only in the case of symmetry the value  $Q_{top}^{(f)}$  becomes integer on important self-consistent fields for every flavour  $f$ . The vacuum complexes exactly repeat the flavour symmetry in their structure (for example, a symmetrical condensate  $\Pi_{f=1}^{N_1} q_R^f \bar{q}_L^f$  or  $\Pi_{f=1}^N \bar{q}_R^f q_L^f$  for the  $SU(N)$ -case). If there is no any flavour symmetry, there are no condensates. But chiral complexes with  $\ln V$  particles corresponding to Eq.(17) ( $\int dp/p \sim \ln V$ ) are present here in vacuum states. We have quantitatively investigated these questions in Refs.[7,9].

Some properties of A and B fields are very similar to instanton fields in QCD [15]:

- i) Dirac determinant  $D(A)$  vanishes on the instanton field:
- ii) Euclidean winding number  $Q_{top} \neq 0$ ;
- iii) an instanton field also necessarily produces massless quark pairs [16];
- iv) total chirality is changed by the instanton field according to Eq.(23).

But in the case of instantons  $D(A)$  vanishes due to the zero mode of the Dirac equation. Therefore, the Green function of a massless quark become s infinite. That changes the quark chirality. The quark functions stay finite in  $B$ -fields of QED<sub>2</sub>. For this reason, the chirality, in fact, appears to be the conserved number. EM-fields can produce in QED<sub>2</sub> only  $q_R\bar{q}_R$  and  $q_L\bar{q}_L$ -pairs, but  $2|Q_{top}|$  of these particles appear immediately in the unobserved vacuum condensate.

The essential fields were studied for QED<sub>2</sub> also in Ref.[12]. The consideration is based on the Euclidean functional integral method. Our consideration performed in the Minkowsky space and the Coulomb gauge to be used indicates an importance of EM field configurations different from those discussed in [17].

### 3 Physical role of the $D(A) \rightarrow 0$ fields in Schwinger model

We show in the present Section that EM-fields where  $D(A) \rightarrow 0$  are responsible for

- A) both the absence of charged particles and for screening of charges;
- B) hadron states to have a definite chirality ( $K = 0$ ) in spite of the existence of the chiral condensate.

As it was stated in Sec.2, Gauss integrals for matrix elements of the  $S(T)$ -operator are evaluated by a substitution of a saddle-point field  $\bar{A}$ . This field depends on positions of particles in initial and final quark configurations. Those are the configurations determining investigated matrix elements. The formula for saddle-point field is obtained in Ref.[10] (Eq.(59) of Ref.[10]). One has to consider only small momenta  $p \ll m$  as the results of Sec.2 witness. In the case of total charge  $Q = 0$  we have the following equation in the  $p \rightarrow 0$  limit:

$$\begin{aligned} A_0(p, t) &= i \frac{g}{m} \frac{R_i(p) + R_f(p)}{| \quad |} p \frac{\cos m(T/2 - t)}{\sin m \cdot T/2} \\ &- i \frac{g}{m} \frac{R_f(p) - R_i(p)}{| \quad |} p \frac{\sin m(T/2 - t)}{\cos m \cdot T/2} . \end{aligned} \quad (24)$$

$R_i(p)$  and  $R_f(p)$  are the sources depending on initial and final quark configurations:

$$R_i(p) = \sum_k \Theta(-p) \left( e^{-ip\tilde{x}_k} - e^{-ip\tilde{x}'_k} \right) + \Theta(+p) \left( e^{-ip\tilde{y}_k} - e^{-ip\tilde{y}'_k} \right) , \quad (25)$$

$$R_f(p) = \sum_k \Theta(-p) \left( e^{-ipx_k} - e^{-ipx'_k} \right) + \Theta(p) \left( e^{-ipy_k} - e^{-ipy'_k} \right) ,$$

where  $\tilde{x}, \tilde{x}' (\tilde{y}, \tilde{y}')$  are coordinates of  $R(L)$  quarks and antiquarks in an initial state;  $x, x' (y, y')$  are those in a final state. The second term in Eq.(24) is inessential both for Green functions and  $D(A)$  because it vanishes being integrated over  $t$ . So it can be omitted.

If charged particles exist, then charges at the large distance  $r$  (for example,  $q$  and  $\bar{q}$ ) inevitably would contribute to the wave function of two charged particles. Such  $S(T)$  matrix element will be independent on  $r$  within the  $r \ll T$  time interval, i.e. within the region

$$|x_0 - \tilde{x}_0| \sim |x'_0 - \tilde{x}'_0| \sim T \ll r, \quad |x_0 - x'_0| \sim r . \quad (26)$$

But the moving ( $\tilde{x}_0 \rightarrow x_0$ )  $R$ -quark contributes to the matrix element with the field strength:

$$E^{(q)}(p, t) = \frac{g}{m} \left[ \Theta(p) e^{-ip\tilde{x}_0} - \Theta(-p) e^{-ipx_0} \right] \frac{\cos m(T/2 - t)}{\sin m \cdot T/2} \quad (27)$$

or in the coordinate space:

$$E^{(q)}(x, t) = \frac{ig}{2\pi m} \left[ \frac{1}{x - \tilde{x}_0 + i0} + \frac{1}{x - x_0 - i0} \right] \frac{\cos m(T/2 - t)}{\sin m \cdot T/2} . \quad (28)$$

This is just the field of the A-type. To obtain the field strength for an  $R$ -antiquark we take  $x_0 \rightarrow x'_0$ ,  $\tilde{x}_0 \rightarrow \tilde{x}'_0$  and change the general sign in Eqs. (27) and (28).

Substituting Eq.(27) into Eqs. (7),(9) we obtain that the  $q_R \bar{q}_R$  configuration, contributing to  $S(T)$  as (Eqs.(3),(9)):

$$b_R(\tilde{x}'_0) a_R(\tilde{x}_0) G_R^{(T)}(T, \tilde{x}'_0; 0, \tilde{x}_0) D(\bar{A}^{(q+\bar{q})}) G_R^{(T)}(T, x_0; 0, x'_0) a_R^+(x_0) b_R^+(x'_0) \quad (29)$$

decreases at  $r \rightarrow \infty$  as  $r^{-2}$  ( $D(A^{(q)}) \sim r^2$ ,  $G \sim r^{-2}$ ). Let us note that  $D(\bar{A}^{(q)})$  does not vanish but increases with  $r \rightarrow \infty$ . This is a direct consequence that  $E^{(q)}$  is a purely imaginary value at  $(x_0 - \tilde{x}_0)$  of Eq.(28). However it is shown in the Appendix that the result  $r^{-2}$  can be obtained also by integrating over the following real fields of the A-type:

$$E(p, t) = i\beta \left[ \Theta(p)e^{-ip\tilde{x}_0} - \Theta(-p)e^{-ipx_0} \right] \frac{\cos m(T/2 - t)}{\sin m \cdot T/2}, \quad (30)$$

where  $\beta$  is a real parameter. The  $r$  dependence in the discussed amplitude is due only the integration over the  $\beta$  parameter in (30). In this case we have that  $D(A) \sim 1/r^{2\beta^2}$  and  $G_{R,L} \sim 1/r^{2i\beta}$ , the Jacobian being  $\sim \sqrt{\ln r}$ . Therefore, the integration over fields, Eq.(30), gives the same result as Eq.(28):

$$\int_0^\infty d\beta \sqrt{\ln r} \frac{1}{r^{2\beta^2}} \frac{1}{r^{4i\beta}} \sim \frac{1}{r^2} \quad \text{at } r \rightarrow \infty. \quad (31)$$

Thus, the screening mechanism is based on the fields of the class A with  $D(A) \rightarrow 0$ . It means that  $S(T)$  contains only configurations with  $q$  and  $\bar{q}$  near each other. Additional particles should be present in configurations if some quark situated far from some antiquark. Their role is to compensate charges of distant  $q\bar{q}$  particles. We have seen in the previous section that the smallness of  $D(A)$  can be compensated by a contribution of quarks with small momenta  $p \sim 1/r$ . One can demonstrate this phenomena for the considered case. For this aim let us substitute the EM field, Eq.(27), into the factor

$$\int dx dx' G_R^{(T)}(x, T, x', T) a_R^+(x) b_R^+(x') \quad (32)$$

representing in  $S(T)$  an additional  $q_R \bar{q}_R$ -pair in the final state. If the pair creation happens near the quark  $x_0 \sim \tilde{x}_0$ , we can neglect the influence of the antiquark  $(x'_0 - \tilde{x}'_0)$  field:  $x \gg |x - x_0| - |x' - \tilde{x}_0| \sim |x_0 - \tilde{x}_0| \sim T_0$ . Using (7), we obtain the Green function  $G_R^T(x, T; x', T)$  for the field (27). In the region  $|x - x_0| \gg |x' - x_0| \sim T$  this Green function becomes:

$$G_R^{(T)}(T, x; T, x') = \frac{1}{2\pi i} \exp \left[ \int_0^T dt_1 \frac{\cos m(T/2 - t_1)}{\sin m \cdot T/2} \ln \frac{1}{T - t_1 - x' + x_0 - i0} \right]. \quad (33)$$

In evaluating Eq.(33) we use the following formula

$$\begin{aligned} \int G_R^{(0)}(x, T; x_1, t_1) dx_1 dt_1 A_0(x_1, t_1) &= \int \frac{dx_1 dt_1}{2\pi i} \frac{A_0(x_1, t_1)}{T - x + x_1 - t_1 - i0} \\ &= - \int \frac{dx_1 dt_1}{2\pi i} \ln(T - x + x_1 - t_1) E(x_1, t_1). \end{aligned} \quad (34)$$

The Green function (33) does not depend on the quark coordinate  $x$ . It depends only on the antiquark coordinate  $x'$ . Thus the additional  $q_R \bar{q}_R$ -pair in the wave function (29) represents a quark with the zero momentum ( $\int a_R^+(x) dx = a_R^+(p=0)$ ) and an antiquark with

a finite momentum determined by the field strength  $E^{(q)}(x, t)$ . The quark  $x$  is situated far from the original one ( $x_0 \sim \tilde{x}_0$ ), while the antiquark  $x'$  is near  $x_0$ , and, hence, its charge participates in the screening. The same effect happens with  $L$ -Green functions.

Thus A-fields prohibit the existence of charged configurations and inevitably provide the last ones with local screening charges.

Now we proceed with the B-type fields. They maintain the chirality conservation of any quark configurations. In order to prove the statement let us consider an appearance of an additional pair  $a_R^+(x_0)b_L^+(y'_0)$  in the final state, i.e. the changing of chirality by two. Eq.(24) shows that such a pair produces the B-type field:

$$E^{(c)}(p, t) = -\frac{g}{m} \left[ \Theta(-p)e^{-ipx_0} + \Theta(p)e^{-ipy_0} \right] \frac{\cos m(T/2 - t)}{\sin m(T/2)}, \quad (35)$$

$$Q_{top}^{(c)}(T) = -\frac{g}{2\pi} \int_0^T dt E^{(c)}(0, t) = 1. \quad (35')$$

The B-field nullifies  $D(A)$  and prohibits the suggested process. It generates additional  $qq$ -pairs. Indeed, let us consider the factor representing two such pairs:  $a_R^+(x)b_R^+(x')$  and  $a_L^+(y)b_L^+(y')$ . The wave function acquires the factor

$$\int dx dx' dy dy' a_R^+(x) G_R^{(T)}(x, T; x', T) b_R^+(x') a_L^+(y) G_L^{(T)}(y, T; y', T) b_L^+(y'). \quad (36)$$

For the fields (35), the Green functions  $G_R$  and  $G_L$  do not depend on  $x$  and on  $y'$  respectively. Thus  $E^{(c)}(p, t)$  produces a chiral pair  $\bar{q}_R q_L$  near the original chiral pair and  $q_R \bar{q}_L$ -particles with small momentum to be far from the region  $E^{(c)} \neq 0$ . The first chiral pair compensates the change of the local chirality carried by the original one. The existence of the chiral condensate seems to be very natural when the creation of chiral pairs with vanishing total momenta appear. As it was explained in Sec.2, condensates really arise in QED<sub>2</sub> model when  $Q_{top}$  is an integer number for all essential B-fields. Such conditions exist in the Schwinger model (see Eq.(35')), and in the many electron model [4,6] with some symmetry between flavours.

Let us note that B-type fields prohibit the change of chirality, but states with the definite chirality can exist. For example, the configuration with a  $q_R \bar{q}_L$  pair both in the initial and in the final states produces the field:

$$E(x, t) = \frac{ig}{2\pi m} \frac{\cos m(T/2 - t)}{\sin m \cdot T/2} \left[ \frac{1}{x - \tilde{x}_0 + i0} - \frac{1}{x - \tilde{y}'_0 - i0} + \frac{1}{x - x_0 - i0} - \frac{1}{x - y'_0 + i0} \right]. \quad (37)$$

For this field  $D(A)$  is non-zero and additional pairs need not be produced. These quark configurations form the chiral condensate in the Schwinger model [7].

To summarize, we see that localized states only with the definite chirality can be stationary in QED<sub>2</sub>. The chirality exchange between them and the vacuum condensate is impossible. As we have already noted that is the property which makes possible the confinement of fermion numbers in QED<sub>2</sub>.

## 4 Space-time picture of charge screening and of the hadronization in physical processes

The production of a chiral quark pair  $q_R \bar{q}_L (\bar{q}_R, q_L)$  by an external source is an analogy of the  $e^+ e^-$  hadron process in QED<sub>2</sub>. The process had been studied in Refs.[5,8,9]. The formulae for EM charge densities  $\rho_{R,L}(x, t)$  for  $R$ - and  $L$ -particles were obtained there to be

$$\rho_{R,L}(x, t) = \pm \left[ \delta(t \mp x) - \frac{m}{2} \frac{t \pm x}{\sqrt{t^2 - x^2}} J_1 \left( m \sqrt{t^2 - x^2} \right) \Theta(t^2 - x^2) \right], \quad (38)$$

$J_1$  being the Bessel function. One can observe in Eq.(38) the leading and the screening charges. The both are situated near the light cone. The screening charge is accumulated in the region  $\Delta x \sim 1/m^2 t$  near the leading particle. After the time  $t \sim p_{in}/m^2$  ( $\Delta x \sim p_{in}^{-1}$  and  $x \sim m^{-1}$  in the restframe) the leading quark starts to interact with the screening ones. The leading quark moves as a free particle till times  $t \leq p_{in}/m^2$  [5,8].

We consider in this section quantitatively all three stages for evolution in time of the parton process: a) neutralization of jets, b) accumulation of partons, c) transition into hadrons. The physics is closely connected with the field generated by a  $q_R \bar{q}_L$  pair, i.e. with the B-type field.

Let us formulate the results.

1.  $R$  and  $L$  jets practically do not effect with each other after the time  $t_0 \sim m^{-1}$ , when particles to be already generated neutralize total charges and chiralities of each jet.
2. The neutralizing charges appear as a chiral pair  $(\bar{q}_R q_L)$  with momenta  $p \sim m$ . This effect is the result of B-field action.
3. Quarks of large momenta  $p \gg m$  are accumulated in almost neutral systems ( $t > t_0$ ). They exist only at the light cone region. They softly turn into hadrons, which slow down  $\sim p$ , the hadronization time is  $t \simeq p/m^2$ .
4. Inside the light cone there are only quarks to have been organized into hadrons. In the region between charges there are no any string of quarks-antiquarks.

The screening of total jet charges ( $q_R$  and  $\bar{q}_L$ ) and the transition into neutral hadrons require to change chirality of the  $q_R \bar{q}_L$ -pair ( $K = 2$ ). As we know (Sec.3) the field of B-type must appear with such effect. Indeed, calculating the field strength  $E(x, t)$  for the distribution of Eq.(38) we obtain

$$E(x, t) = g J_0 \left( \sqrt{t^2 - x^2} \right) \Theta(t^2 - x^2). \quad (39)$$

It is the field of the B-type since at  $p \rightarrow 0$

$$E(p, t) = 2g \frac{\sin mt}{m}. \quad (40)$$

The corresponding shift of chirality is equal to

$$K(t) - K(0) = 2Q_{top}(t) = 2(\cos mt - 1). \quad (41)$$

The chirality changing (41) implies the creation of two kinds of quark pairs by the B-field (see Sec. 2–3).

1) One additional  $\bar{q}_R q_L$  pair with the momentum  $p \sim m$ . These particles compensate total charges and chiralities of  $R$  and jets. This pair exists after  $t_0 \sim 1/m$ .

2) One  $\bar{q}_R q_L$  pair with the total momentum  $p \sim 1/t$ . It is expanded always over the whole causal region ( $2t$ ). This pair appears simultaneously with the first one, as chirality of local QED<sub>2</sub> systems can be changed only on the time of fluctuations ( $\sim 1/m$ ). Chirality oscillations mean a fluctuative exchange between just this pair and the condensate. The pair joins the condensate at  $t \rightarrow \infty$ .

The latter pair has no connection with the parton accumulation and transitions into hadrons. The both processes generate independent and neutral ( $Q = 0$ ,  $K = 0$ ) compact  $R, L$  packets. The quarks of the first  $\bar{q}_R q_L$  pair participate in the hadron production. They form hadrons with  $p \sim m$  at the time  $t \sim t_0 \sim 1/m$ .

To prove the above assertions, we investigate the time dependence of the number of quarks produced by the field, Eq.(39), in the considered process. The strict calculations by means of the bosonization method gives [9] the answer very similar to Eq.(13). The density of the right-handed quarks is

$$\begin{aligned} n_R(p, t) &= \langle \Psi(t) \beta_R^+(p) a_R(p) | \Psi(t) \rangle = \\ &= \int \frac{dx dy}{2\pi i} \frac{e^{ip(x-y)}}{y - x - i0} f(x - y) \left[ e^{2\pi i(\alpha(x,t) - \alpha(y,t))} - 1 \right]. \end{aligned} \quad (42)$$

Here,  $\alpha(x, t)$  can be expressed in terms of the field strength  $E(x, t)$ , Eq.(39), by means of Eq.(14). The function

$$f(x - y) = \exp \left[ - \int \frac{dk}{w_k} \left( \frac{w_k - k}{k} \right)^2 \sin^2 \frac{k(x - y)}{2} \right], \quad w_k = \sqrt{k^2 + m^2} \quad (43)$$

determines the mean density of the quarks in the vacuum.

The integrand in Eq.(42) depends on  $\xi = t - x$  and  $\xi' = t - y$ . For  $t \gg t_0 \sim 1/m$  and  $p \leq m$  only the difference  $\xi - \xi'$  is restricted by a rapid decrease of the function  $f(x - y) = f(\xi)$ . The values of  $\xi$  and  $\xi'$  themselves are large ( $-t$ ). In this case the integration in Eq.(14) goes over all possible  $x_1$  and  $t_1$ . Thus, we have

$$\alpha(x, t) = -\frac{g}{2\pi} \Theta(t - x) \int_0^t dt_1 \int g J_0(m) (t_1^2 - x_1^2) \Theta(t_1^2 - x_1^2) dx_1. \quad (44)$$

Applying Eq.(16) and introducing the variables  $\zeta = \xi - \xi'$  and  $\xi'$ , we obtain

$$n_R(p, t) = \int_p^\infty \frac{dx}{2\pi} \int d\zeta, e^{-ik\zeta} f(\zeta) F(\zeta), \quad (45)$$

where

$$F(\zeta) = \zeta \exp[i\pi Q_{top}(t) \varepsilon(\zeta)] 2i \sin 2\pi Q_{top}(t), \quad (46)$$

and  $Q_{top}(t)$  is the same as in Eq.(41). Finally, substituting Eq.(46) in Eq.(45) we get

$$n_R(p, t) = A(p) \sin^2 \pi Q_{top}(t) + B(p) \sin 2\pi Q_{top}(t) , \quad (47)$$

and

$$A(p) = \frac{2}{\pi} \int_0^\infty d\zeta f(\zeta) \sin p\zeta, \quad B(p) = \frac{1}{\pi} \int_0^\infty d\zeta f(\zeta) \cos p\zeta . \quad (48)$$

Equation (47) clearly demonstrates the oscillating nature of the density of quarks produced with small momenta. The first term in Eq.(47) describes production of quarks by the field given by Eq.(39). For  $f(\zeta) = e^{-\delta\zeta}$  ( $\delta = 0$ ) we return to Eq.(17). The second term describes the influence of the field on vacuum quarks. That is the effect which produces oscillations.

$R$  quarks of Eqs.(42)–(48) are bounded into chiral pairs. Since in Eq.(41) there are no restrictions on  $x$  and  $y$  values ( $|x|, |y| \leq t$ ), the pairs are expanded over the whole volume  $\sim 2t$ , i.e. the total momentum of the pair is  $p \sim t^{-1}$ . The momenta of the quarks inside one pair are determined by the integrals in Eq.(48) and are of the order of  $p \sim m$ . The total number of bounded quarks is

$$\int A(p) \frac{dp}{2\pi} \sim \int B(p) \frac{dp}{2\pi} \sim 1. \quad (49)$$

Therefore we have dealt with one pair. The value of  $Q_{top}$  oscillations also confirms this fact.

Let us now consider particles with momenta  $p \gg m$ . They do not participate in transitions into vacuum, but just these particles form hadrons. The last ones with momentum  $P$  are formed during the time

$$t \sim \frac{P}{m^2} . \quad (50)$$

The size of the quark packet, Eq.(38), becomes then equal to the size of such hadrons  $\sim 1/P$ . After the time  $\sim P/m^2$  all quarks with momenta  $p \leq P$  have to be bounded. Equation (42) gives the opportunity to verify this qualitative picture. In some aspects we repeat Ref.[9].

For particles with  $p \gg m$  the value of  $|x - y|$  becomes  $\sim 1/p$ , and  $f(\zeta) \sim f(0) = 1$ . These quarks do not interfere with vacuum quarks. We apply again Eq.(16) and rewrite Eq.(42) in the form ( $\zeta = (t - x)$ ):

$$n_R(p, t) = \int_p^\infty \frac{dx}{2\pi} \left| \int_{-\infty}^{+\infty} e^{-ik\zeta - 2\pi i\alpha(\zeta, t)} d\zeta \right|^2 . \quad (51)$$

Since jets become neutral at  $t \sim 1/m$  we have  $n_R = \bar{n}_R = n_L = \bar{n}_L$ .

In Eq.(14) the essential integration region is near to the light cone:

$$0 < \zeta_1 = t_1 - x_1 < \zeta = t - x \sim \frac{1}{p} . \quad (52)$$

Therefore, quarks with momenta  $p$  are produced only in the region  $1/p$  near the leading particles. Inside the light cone there are quarks with  $p < m^2 t$ . They have already formed hadrons.

According to Eq.(52) the  $t_1^2 - x_1^2$  interval is approximately  $t_1^2 - x_1^2 = 2t_1\zeta_1$ , and  $\alpha$  is

$$\alpha(\zeta, t) = -\frac{m^2}{2} \int dt_1 \int_0^\zeta d\zeta_1 J\left(m\sqrt{2t\zeta_1}\right) = e(\zeta) \left[ J_0\left(m\sqrt{2t\zeta}\right) - 1 \right] , \quad (53)$$

since  $\Theta(t_1^2 - x_1^2)\Theta(\zeta - t_1 + x_1) = \Theta(\zeta_1)\Theta(\zeta - \zeta_1)$ . Eq.(53) shows that jets are independent from each other for  $(t > 1/m)$  because only the field of the  $R$ -jet produce  $R$ -quarks. In the time interval  $m^{-1} \ll t \ll p/m^2$  we can expand the Bessel function in Eq.(53) and get

$$n_R(p, t) = \frac{\pi}{6} \left( \frac{m^2 t}{p} \right)^2 \frac{1}{p}, \quad m^{-1} < t < \frac{p}{m^2} . \quad (54)$$

At times  $t > p/m^2$  the argument of the Bessel function becomes large and we expand the exponential factor in Eq.(51) in powers of  $\alpha(\zeta, t)$ . The integrations over  $k$  and  $\zeta$  give the distribution to be independent of time as follows

$$n_R(p, t) = \frac{2\pi}{p} , \quad t \gg \frac{p}{m^2} . \quad (55)$$

At times  $t \sim p/m^2$  the distribution of Eq.(54) becomes the order of unity. Quarks with the momentum  $p$  begin to turn into hadrons. Eq.(55) represents the distribution of quarks inside hadrons.

Indeed, we have in the parton model

$$n_R(p, t) = \int N(P, t) n_R(P, p) \frac{dP}{2\pi} . \quad (56)$$

Here  $N(P, t)$  is the number of hadrons with the momentum  $P$ . In the process considered this quantity is equal to [5,9]:

$$N(P, t) = \frac{\langle \Psi(t) | A^+(p) A(P) | \Psi(t) \rangle}{\langle \Psi(t) | \Psi(t) \rangle} = \frac{2\pi}{\omega_p} . \quad (57)$$

Here,  $A^\pm(p)$  are hadron operators. The distribution of quarks inside a hadron with the momentum  $P$  is equal to:

$$n_R(P, p) = \langle \Psi_h(P) | a_R^+(p) a_R(p) | \Psi_h(P) \rangle = \frac{2\pi}{P} . \quad (58)$$

We obtain Eq.(58) by means of the parton wave function for a QED<sub>2</sub> hadron. We had found in Refs. [9,10] that

$$\begin{aligned} \Psi_h(P) &= \sqrt{\frac{2\pi}{P}} \int \frac{dk}{2\pi} a_n^+(P-k) b_R^+(k) | \Omega_0 \rangle , \quad P \gg m \\ a_{R,L}(k) | \Omega_0 \rangle &= b_{R,L}(k) | \Omega_0 \rangle = 0 , \quad \text{for } k \gg m . \end{aligned} \quad (59)$$

Substituting Eqs. (57)–(58) in Eq.(56) we obtain the distribution of Eq.(55).

The leading particles lose their energies and turn into hadrons after the time  $t \sim P_{in}/m^2$ . Hence, the most rapid hadrons are produced in the last turn (it is similar to Refs.[5,17]).



## 5 Conclusion

In the present paper it was demonstrated that confinement of the charged states in QED<sub>2</sub> is provided by the field configurations which nullify the Dirac determinant while Green functions stay finite in these fields. It is possible that this confinement mechanism can exist not only in two dimensions. But at present it is unknown whether the fields with similar properties play any role in QCD, in four dimensions. But if the  $D(A) \sim 0$  fields are important also in QCD, the mechanism based on such fields would be very attractive because it naturally provides theory with many phenomenological inviting properties.

In fact, the colour screening in QCD could appear because quark-antiquark create a coloured field providing for the  $D(A)$  determinant to go to zero when the distance between them increases. This field produces (with probability of one) soft quarks which screen the initial colour. These quarks create the baryon number current which could, generally, screen also the triality of the sources.

Certainly, the screening is much more complicated for QCD because of its non-Abelian nature (also we must screen the gluon colour) and probably needs more time than in QED<sub>2</sub>. However in the described scenario the momenta of leading quarks are much more than momenta of the screening ones. So it seems that the time of screening would be less than the time of hadronization. As a consequence hadrons are produced after the time when the screening of colour is over and jets are independent and colourless. Thus the fast hadrons (which are produced the last) would be created quite independently in different jets. Light quarks would play a fundamental role in this confinement mechanism.

## A

The explanation concerning the imaginary saddle field in Eqs.(28),(29) consists in the explicit evaluation of the functional integral. We have for the matrix element considered:

$$S_0 \equiv S_{q\bar{q}}^{q\bar{q}}(T) = \int DA(x) \exp[iS(A)] G_R^{(T)}(T, x'_0; 0, \tilde{x}'_0) D(A) G_R^{(T)}(T, x_0; 0, \tilde{x}_0) . \quad (60)$$

We are interested only in components of  $A$  with very small  $p$ , these ones are important for the dependence on the distance  $r$  between  $q$  and  $\bar{q}$  when  $r \rightarrow \infty$ . Then we have

$$S(A) = \int_0^{P_0} \frac{dp}{2} \left\{ \frac{im^2|p|}{4} a(p)a(-p) + ig[R_1(p) + iR_2(p)]a(-p) \right\} + S(p \gg p_0) , \quad (61)$$

where

$$p_0 \ll m, \quad a(p) = \int_0^T A_0(p, t) dt . \quad (A.2')$$

We neglect in Eq.(A.2) the kinetic energy of the small components:  $\int_0^{P_0} p^2 A_0^2(p, t) dp / 2\pi$ . We take the sources from Eq.(25) and rewrite down them for the  $q\bar{q}$ -case in the following way

$$R_i(p) + R_f(p) = R_1(p) + R_2(p) . \quad (62)$$

$R_1(p) = R_1^*(-p)$  and  $R_2(p) = -R_2^*(-p)$  are real and imaginary parts of the sources.

The integral can be easily transformed to

$$S_0 = Z\lambda \int d\beta e^{ig\lambda\beta} \int Da(r) \delta \left( \int R_1(p) a(p) \frac{dp}{2\pi} - \beta\lambda \right) \exp \left\{ i \int \frac{dp}{2\pi} \left[ \frac{im^2|p|}{4} a(p)a(-p) + igR_2(p)a(-p) \right] \right\} G_R G_R D . \quad (63)$$

Here  $Z$  includes all contributions independent on  $r$ , and

$$\lambda = \int \frac{R_1(p)R_1(-p)}{|p|} \frac{dp}{2\pi} \sim \ln r . \quad (64)$$

Let us change the integration variables according to

$$a(p) = \beta \frac{R_1(p)}{p} + \tilde{a}(p) = a_0(\beta, p) + \tilde{a}(p) . \quad (65)$$

We obtain

$$S_0 = Z\lambda \int d\beta D(a_0) G_R(a_0) G_R(a_0) \cdot I \quad (66)$$

$$I = \int D\tilde{a}(p) \delta \left( \int R_1 \tilde{a} \frac{dp}{2\pi} \right) \exp \left[ - \int \frac{dp}{2\pi} \frac{m^2|p|}{4} \tilde{a}(p) \tilde{a}(-p) \right] .$$

The dependence on  $R_2$  is inessential for  $r \rightarrow \infty$ . Thus the integration in Eq.(47) goes over real fields and these fields are of the A class. Practically we finish the proof of our assertion in the text. The further evaluation of  $I$  is straightforward.

$$\begin{aligned} I &= \int D\tilde{a} \int \frac{d\zeta}{2} \exp \left[ - \int \frac{dp}{2\pi} \frac{m^2|p|}{4} \tilde{a}(p) \tilde{a}(-p) + i\zeta R_1(p) \tilde{a}(-p) \right] \\ &= \int_{-\infty}^{+\infty} \frac{d\zeta}{2\pi} \exp \left[ - \frac{\zeta^2}{m^2} \lambda \right] \sim \frac{1}{\sqrt{\ln r}} . \end{aligned} \quad (67)$$

Substituting all factors in Eq.(A.7) we get the result of the text. Obviously, the saddle point for integration over  $\beta$  is imaginary:  $\beta = i$ , and we return to the field of Eq.(22).

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